NORTH AMERICAN PLATE ROTATING ABOUT EULER POLE (GPS DATA)

Directions follow small circles

Rates increase as sine of angular distance from pole

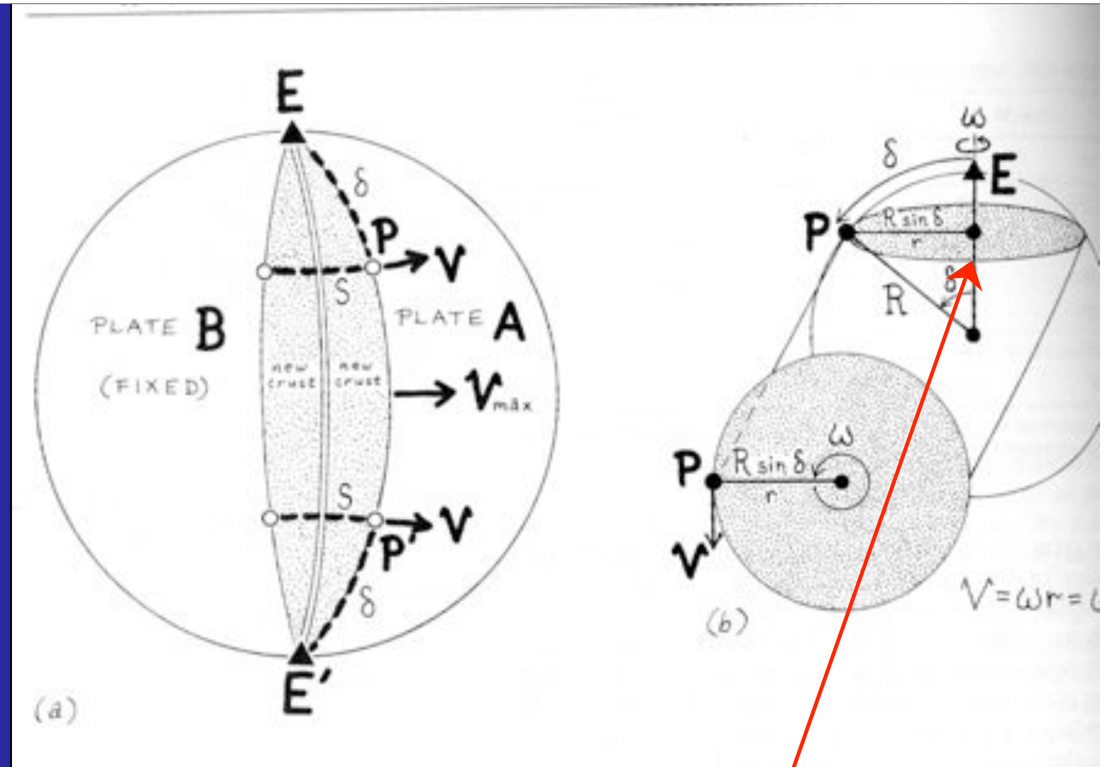
Rotation and angular velocity

Rotation:

change in radial position vector.

Theory of infinitesimal rotations:

dr = r'-r .



Euler Pole &
Rotation vector

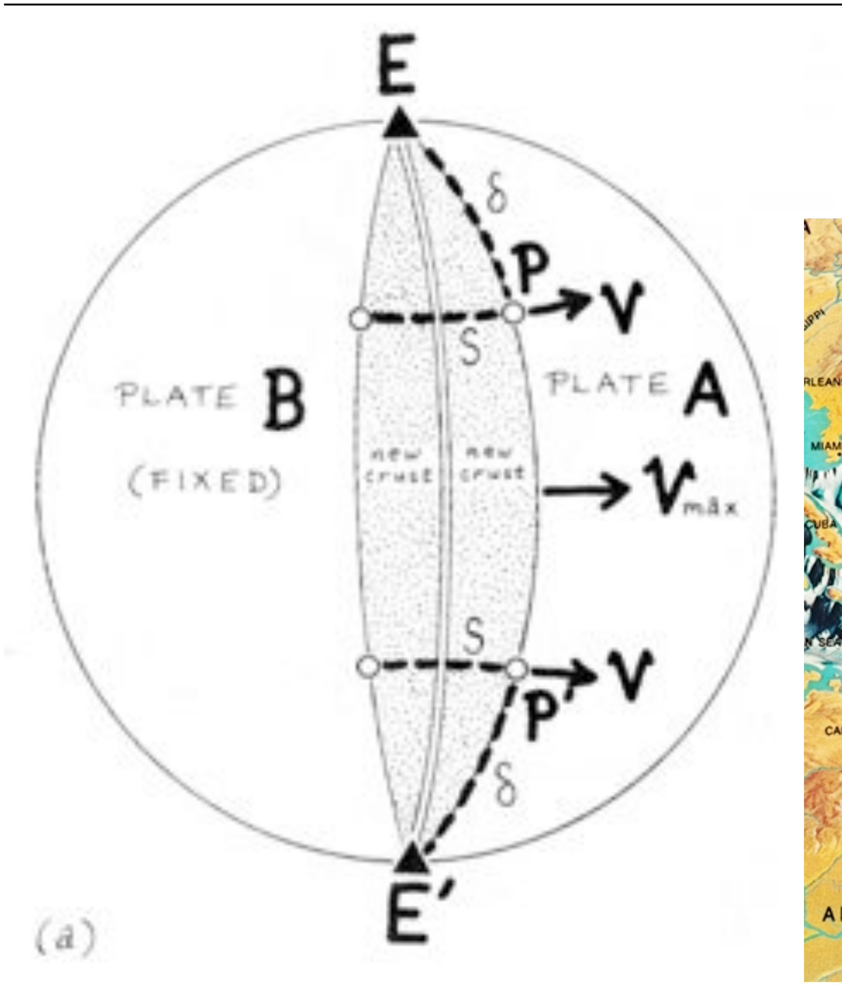
Euler's Fixed Point Theorem:

A change in position can be described by rotation about an axis.

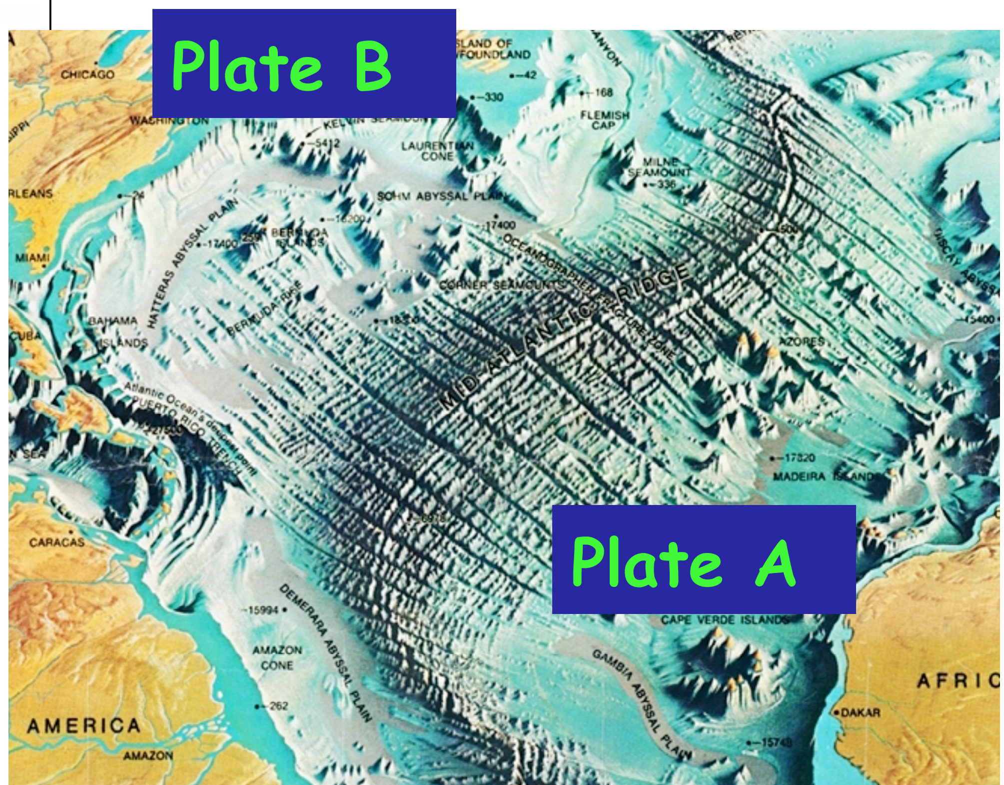
Rotation axis goes through the center of the Earth.

Rotation axis intersects Earth's surface at the *pole of rotation.*

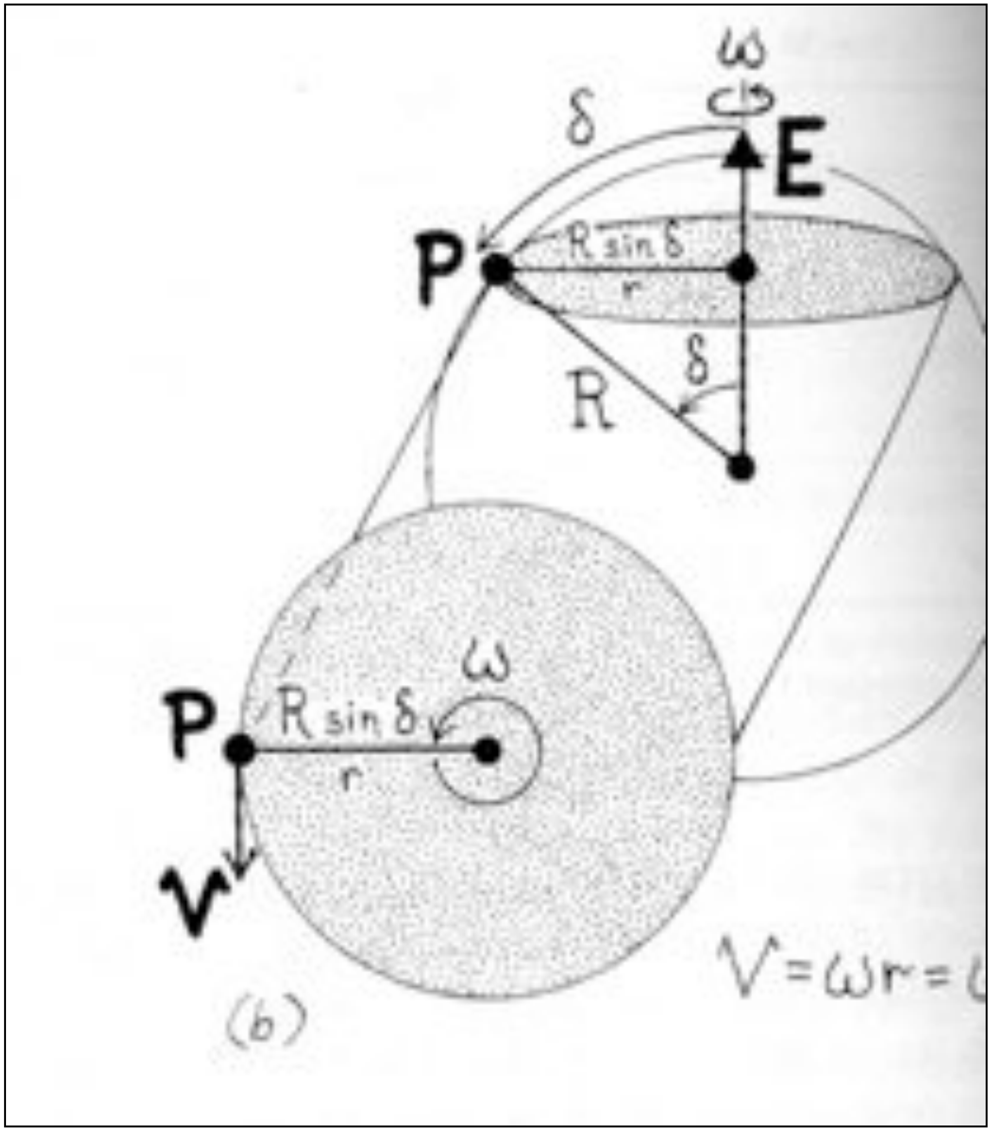
Tectonics on a sphere requires that we use **Spherical Polar Coordinates** and angular velocities



($\omega = v/r$ and $r = R \sin\delta$ where R is the radius of the Earth).



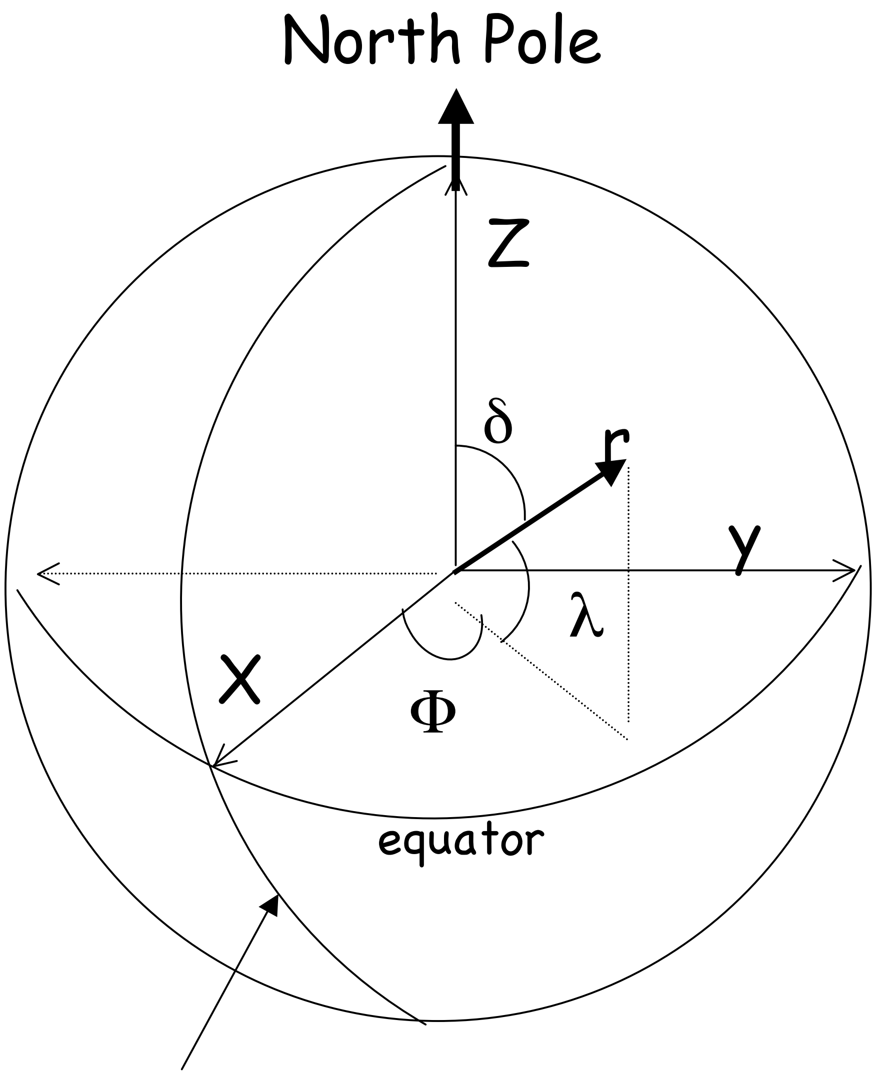
Tectonics on a sphere requires that we use **Spherical Polar Coordinates** and angular velocities ($\omega = v/r$ and $r = R \sin\delta$ where R is the radius of the Earth).



the magnitude of v:

$v = \omega R \sin\delta$

Spherical Polar Coordinates and angular velocities



SPC

$R \equiv |\mathbf{r}| = 6371$ km

$\delta = 90 - \lambda$

$\delta \equiv$ co-latitude

$\lambda \equiv$ latitude

$\phi \equiv$ longitude

+E (0-180°)

- W (180-360°)

Cartesian from SPC

(components of the r vector)

r

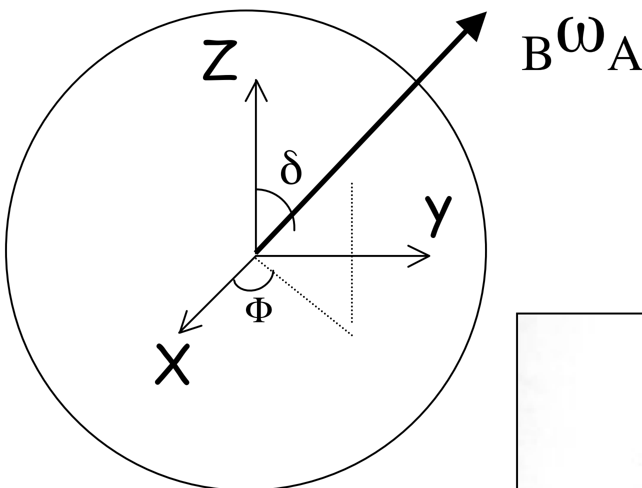
$\mathbf{r}_x = R \sin\delta \cos\phi$

$\mathbf{r}_y = R \sin\delta \sin\phi$

$\mathbf{r}_z = R \cos\delta$

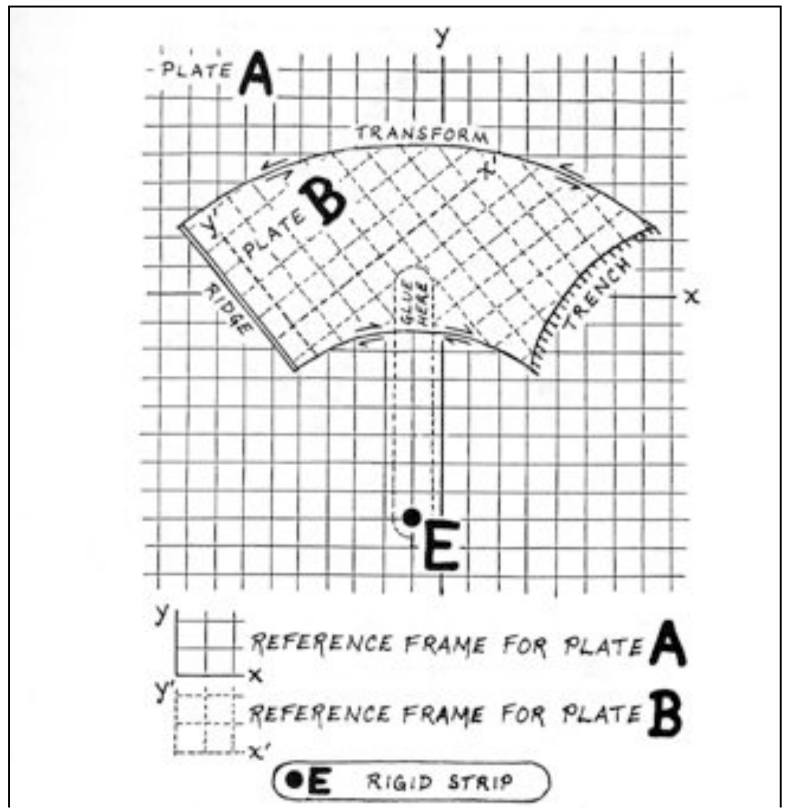
Angular Velocity Vector

The **rotation axis** intersects Earth's surface at the *pole of rotation.*



Components of the angular velocity vector ω

The three components are
1) magnitude,
2) pole co-lat,
3) pole longitude.



Angular Velocity Vector

The **rotation axis** intersects Earth's surface at the *pole of rotation.*

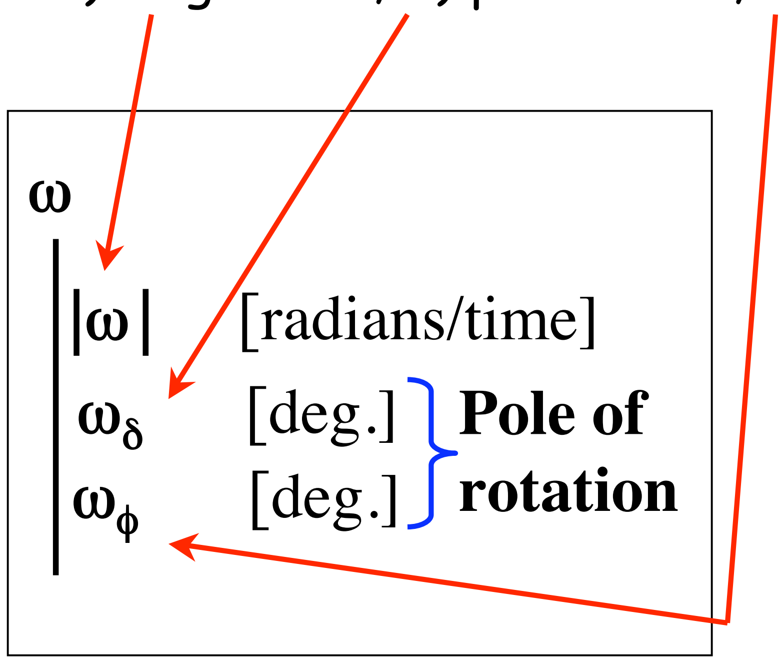
Sign convention for **positive pole** and **negative pole.**

Right hand rule -> ccw positive.

Components of the angular velocity vector ω

- Note: they are not the same as in Cartesian coords.

The three components are 1) magnitude, 2) pole co-lat, 3) pole longitude.



Angular Velocity Vector

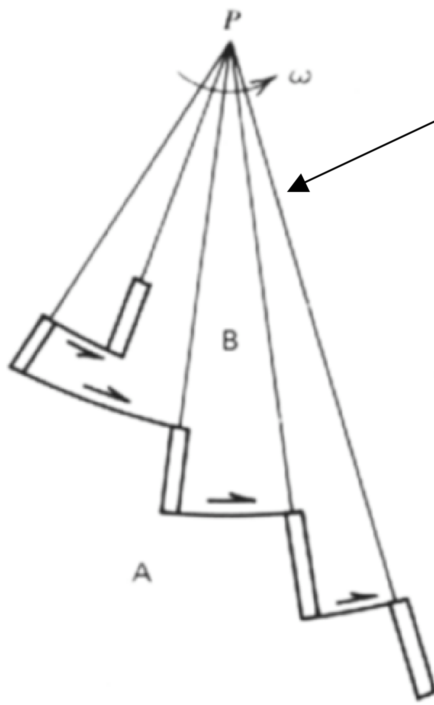
Sign convention for **positive pole** and **negative pole**. **Right hand rule. ccw positive.**

Ideal Plate Tectonics

the magnitude of v: $v = \omega R \sin\Delta$

Transform faults are along **small circles** to pole of rotation

Ridge segments are parallel to **great circles**.



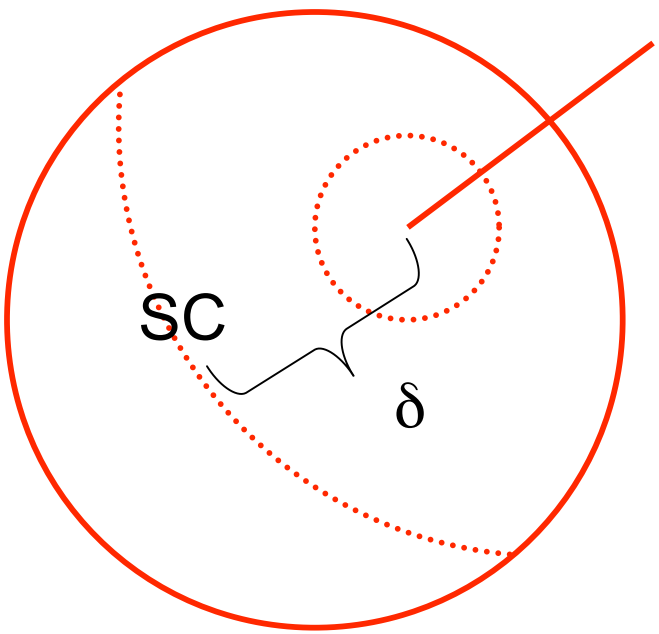
Δ is the length, in degrees, of these lines

Review: we know that $_A\omega_B = -{}_B\omega_A$. If $_A\omega_B$ is [1.5°/Ma, 40.8°, 282.1°]. What is $_B\omega_A$?

Example: Angular Velocity and Linear Velocity

- Find the linear velocity at State College due to rotation around the North Pole

the magnitude of v: $v = \omega R \sin\delta$



$|\omega| = 2\pi$ rad/day (rotation rate)

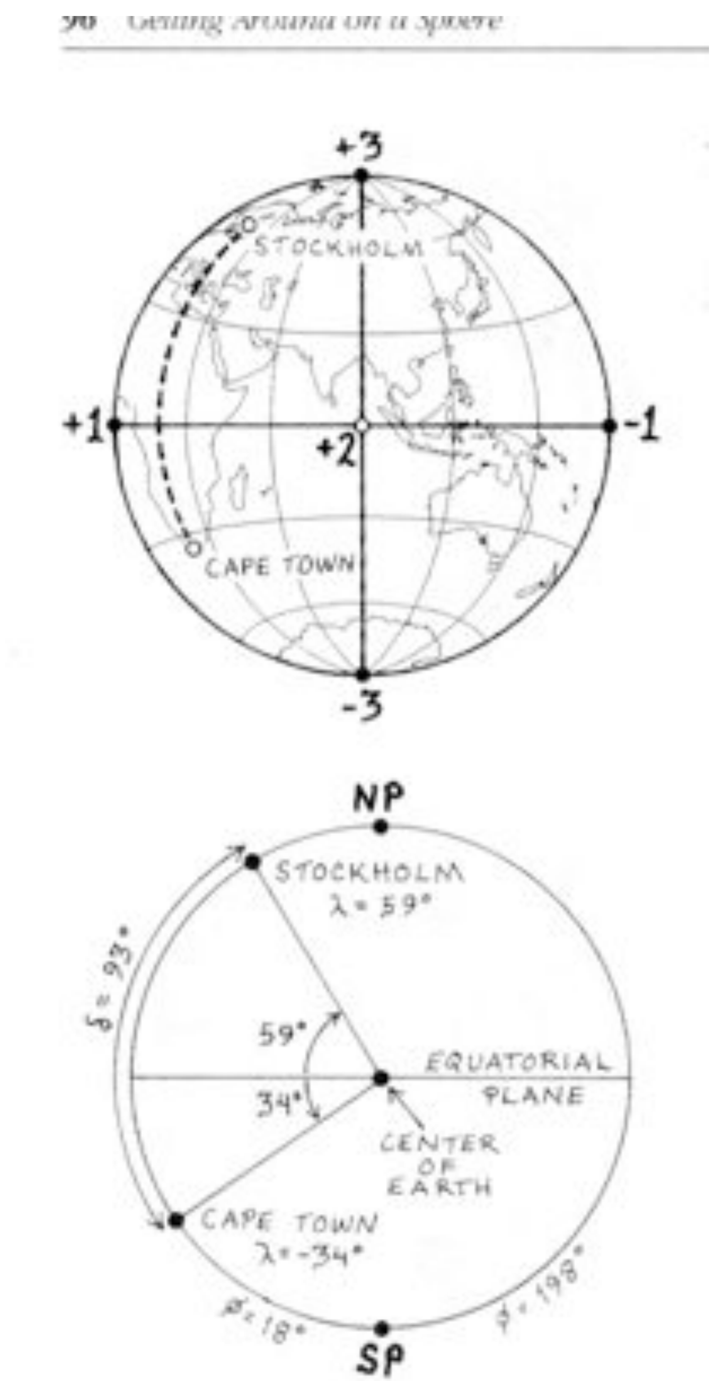
$R = 6371$ km

$\delta = 90 - 40.8 = 49.2°$

$v = 2\ \pi$/day 6371 km sin(49.2°)
= 40,030km * sin(49.2°)
= 30,303 km/d = 0.351 km/s = 758 m/hr

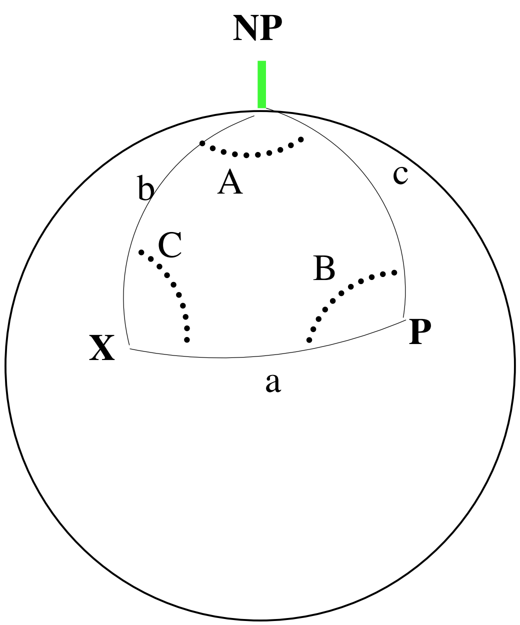
Example:
Find distances on a sphere; use lat, long, and δ (note that δ is sometimes written as Δ)

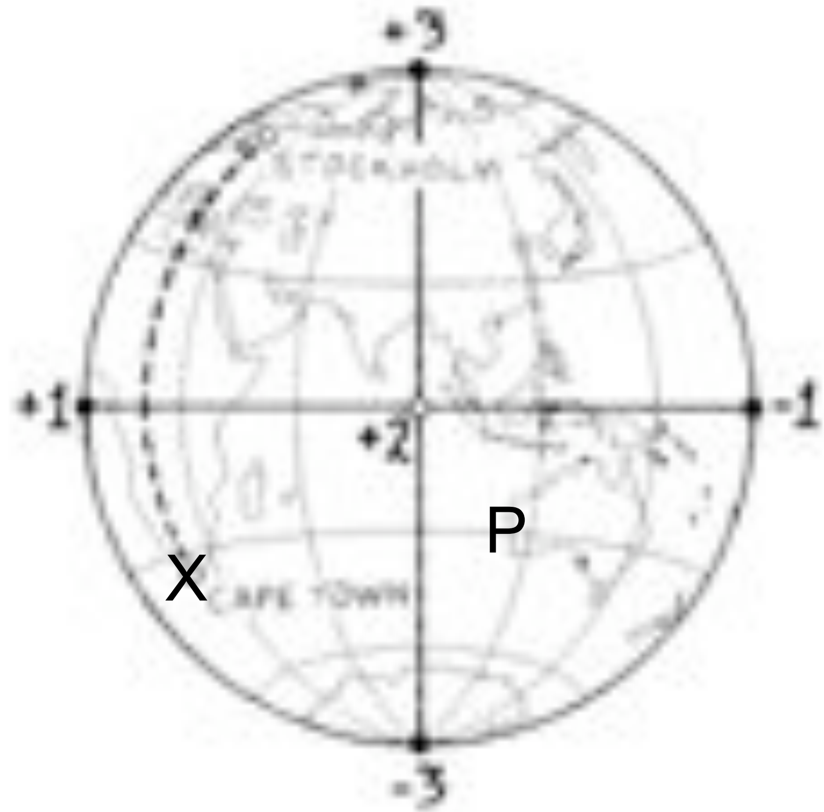
In this example, the locations are along the same line of longitude. Therefore, δ is just the latitude of Stockholm plus the latitude of Cape Town



Example:
More general (harder) case of finding distances on a sphere; use lat long and Δ

What if the two locations are not on the same line of longitude?

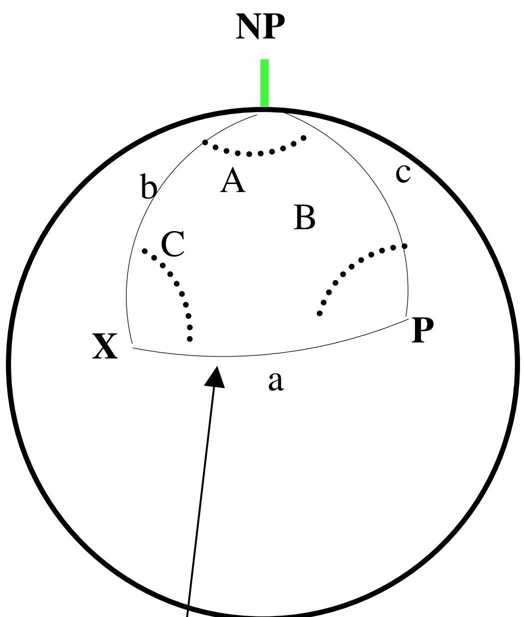




What if we needed the distance from Cape Town to Perth, Australia?

Example:
More general (harder) case of finding distances on a sphere; use lat long and Δ

Use Spherical Geometry



these arcs are segments of great circles

$A = \phi_p - \phi_x$, $b = \delta_x$, $c = \delta_p$, $a = \Delta$

Use spherical trig identity

$\text{Cos } a = \cos b \cos c + \sin b \sin c \cos A$

In our notation:

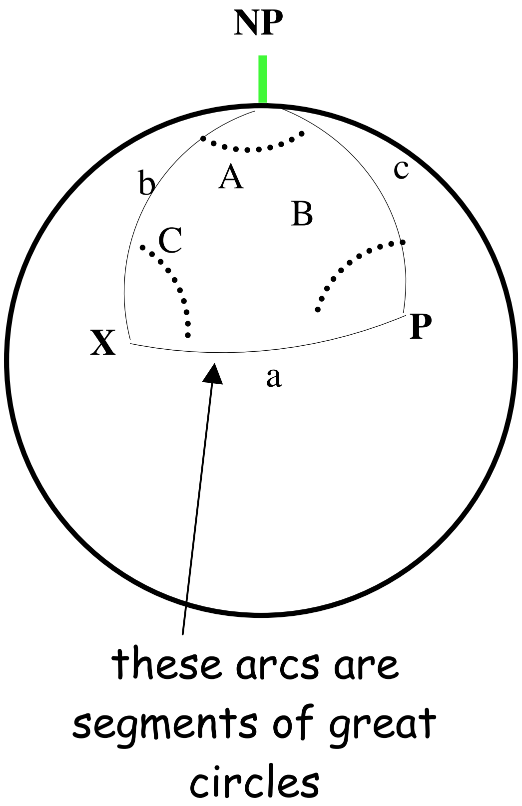
$\text{Cos } \Delta = \cos\delta_x \cos\delta_p + \sin\delta_x \sin\delta_p \cos(\phi_p - \phi_x)$

Also:

$\sin a / \sin A = \sin c / \sin C$

$C = \sin^{-1} [\sin \delta_p \sin(\phi_p - \phi_x) / \sin a]$

But be aware of sign ambiguity for $\sin^{-1}$



Velocity of the North American Plate relative to the Pacific Plate is given by the rotation pole at: 48.7° N 78.2° W and angular velocity 7.8e-7 deg/year
A point on the Pacific plate near Parkfield California, which is at 35.9° N 120.5° W, is moving at 47.8 mm/yr relative to the rest of North America.

To calculate the velocity at Parkfield CA, we need
1) the angular distance between Parkfield and the rotation pole.
and 2) the relation $V = \omega R \sin \Delta$

We can use this equation:

$$\cos \Delta = \cos\theta_x \cos\theta_p + \sin\theta_x \sin\theta_p \cos(\phi_p - \phi_x)$$

We have $\theta x = 90-35.9 = 54.1$, $\theta p = 90-48.7 = 41.3$ and $\phi p - \phi x = (-78.2) - (-120.5) = 42.3$

Plugging in gives $\Delta = 33.28°$

Then we can get V as: 7.8e-7 * (π/180) * 6371e3 * sin(33.28) = **0.0476 m/yr**